

# Having It Both Ways: Distinguishable Yet Phase-Coherent Mixtures of Bose-Einstein Condensates

E. A. Cornell, D. S. Hall, M. R. Matthews, and C. E. Wieman

*JILA, National Institute of Standards and Technology  
and Department of Physics, University of Colorado, Boulder, Colorado*

*We have begun a series of experiments on mixed bosonic quantum fluids. Our system is mixed Bose-Einstein condensates in dilute Rb-87. By simultaneously trapping the atoms in two different hyperfine states, we are able to study the dynamics of component separation and of the relative quantum phase of two interpenetrating condensates. Population can be converted from one state to the other at a rate that is sensitive to the relative quantum phase.*  
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## 1. INTRODUCTION

The observation of Bose-Einstein condensation (BEC) in a dilute gas<sup>1-3</sup> has led to a new generation of experiments in quantum fluids. Historically, in the study of quantum fluids, some of the most intriguing behavior has been found in the behavior of fluid mixtures. We have thus been motivated to pursue a series of mixed-condensate experiments in Rubidium-87.<sup>4-7</sup> The experimental tools and the theoretical world view of atomic physics are almost entirely disjoint from those of traditional low-temperature physics, and this lack of overlap has led to some confusion. The purpose of this paper is to review some of our early results on mixed condensates, and to provide a qualitative exegesis of the theoretical and experimental techniques that underpin our work. Of necessity we have omitted here much of the detail provided in the original papers.<sup>4-7</sup> The MIT BEC group also has begun experiments in mixed condensates.<sup>8</sup>

The experiments described here all begin with a sample of approximately  $5 \times 10^5$  spin-aligned, Bose-condensed Rb-87 atoms at a temperature

of less than 50 nK, confined in a 3-d, harmonic magnetic potential at density  $1 \times 10^{14}/\text{cm}^3$ . The number density of noncondensed atoms is a factor of 30 or more smaller. In this paper we will skip the details of refrigeration — the technique is a hybrid method that combines many of the notable advances<sup>9</sup> in atom cooling from the 1980s: laser cooling and trapping, magnetic trapping, and evaporative cooling. The synthesis of these various approaches was begun at JILA from 1989–1992,<sup>10,11</sup> but it was only after considerable additional development<sup>12–15</sup> that, in 1995, the parts worked well enough together to permit the first observation of BEC in dilute atomic gases.<sup>1–3</sup> In the current version of our apparatus, the cooling process is automated and reliable, much like a (well-behaved) dilution refrigerator. A freshly prepared condensate is available for study about once per minute. The isolation of the sample from the 300 K environment (less than 1 cm away) is excellent — thermal equilibration times with the environment are on the order of centuries — but residual heating and the collisional decay of the condensate limit the duration of an experiment on any given condensate to a few seconds.

The ground state of an Rb-87 atom is split in a magnetic field into eight levels by hyperfine and Zeeman interactions (Fig. 1). The states labelled  $|1\rangle$ ,  $|2\rangle$ , and  $|3\rangle$  can be trapped at a local minimum in the magnetic field; it is with states  $|1\rangle$  and  $|2\rangle$  ( $|F=1, m=-1\rangle$ , and  $|F=2, m=1\rangle$ , respectively) that we have performed most of our mixed-fluids experiments.<sup>17,18</sup> The other relevant internal structure of the Rb-87 atom is a set of excited-state levels 1.6 eV above the ground state (Fig. 1). We probe the spatial distribution of the atoms by imaging the absorption on an optical transition to these levels. The natural 6 MHz linewidth of the optical transition is much less than the energy splitting between, for instance, states  $|1\rangle$  and  $|2\rangle$ . With the correct choice of laser frequency, we can image the spatial distribution of either state  $|1\rangle$  or of state  $|2\rangle$ , or, if we prefer, of the combined density of the two states.<sup>5</sup>

The paradoxical aspect of the Rb-87 mixed-condensate system is that it combines two traits that are usually thought of as mutually exclusive: (i) the  $|1\rangle$  and  $|2\rangle$  components of the condensate are distinguishable, in the usual senses of the term — the components do not at experimental energy scales spontaneously interconvert, and the components can be selectively detected and imaged; and (ii) the components possess a measurable relative quantum phase, which evolves at a rate proportional to the difference in chemical potential between the two fluids, and which can, under the right circumstances, be read out, much as the current across a Josephson junction reads out the relative quantum phase across it. Interconversion (and phase read-out) *can* be driven via a stimulated process and therefore can be turned on and off. The system thus bears considerable resemblance to the idealized

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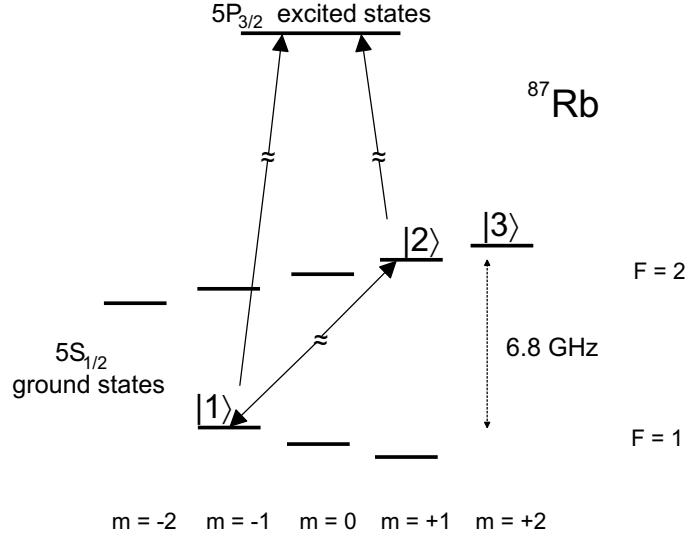


Fig. 1. The relevant hyperfine and Zeeman structure of Rb-87 in a magnetic field. The atoms are moved coherently from state  $|1\rangle$  to state  $|2\rangle$  with a microwave pulse.<sup>16</sup> Imaging is accomplished via absorption in optical transitions to the excited states.

pair of condensates linked by a removable Josephson junction that Leggett<sup>19</sup> and others have envisioned in various thought experiments.

In Section 2 we will discuss distinguishable-fluid experiments; in Section 3 we discuss simple phase-coherent experiments; in Section 4 we discuss experiments that combine the two aspects; and in Section 5 we mention various future experiments with weak coupling.

## 2. INTERPENETRATING, DISTINGUISHABLE FLUIDS

Species  $|1\rangle$  and  $|2\rangle$  meet both usual criteria for calling fluids “distinguishable.” One can in the literal sense distinguish one density distribution from the other, simply by adjusting the frequency of the probe laser so that one species or the other casts its shadow onto our imaging CCD array (Fig. 1). This measurement, as performed in Refs. 5–7, is destructive, but as has been shown<sup>20</sup> it needn’t be so. In any case the fact that we can prepare condensates with a high repetition rate and a large degree of reproducibility means that the destructive aspect of the imaging is not very important to us: we observe the time-evolution of the clouds simply by repeating the experiment many times with increasing dwell-times. The second sense of “distinguishable” is that particles do not spontaneously interconvert. This

requirement is enforced in our system by the enormous difference in internal energies between the two states. In the absence of applied electromagnetic fields, the 6.8 GHz hyperfine energy (Fig. 1) is a million times larger than any other energy scale of the condensate system. The energy released by a *single* atom converting from state  $|2\rangle$  to state  $|1\rangle$ , if that energy were to be distributed thermally through our sample, is sufficient to drive the *entire sample* out of the Bose-condensed state. The fact that we don't observe our condensates melting during our measurements is a guarantee that spontaneous interconversion is not a factor in the system.

The positions of the two fluids are determined by their respective confining magnetic potentials. The magnetic moments of the two states are nominally the same, but due to various small effects — gravity, the nuclear magnetic moment, the onset of nonlinearity in the Zeeman shifts, and subtle dynamical effects<sup>15,21</sup> of our magnetic trap — the location of the two minima of the confining potentials  $V_1(r)$  and  $V_2(r)$  can be adjusted to be either exactly coincident or slightly offset. For the work described here, the confining potentials are axially symmetric and harmonic, with single-particle oscillation frequencies of  $\omega_z/2\pi = 60$  Hz, and  $\omega_r/2\pi = 21$  Hz. With the spatial offset between the two potentials set to zero, and in the absence of interspecies interactions, the two fluids would be completely interpenetrating. In fact, inter- (and intra-) species interactions are a dominant factor in determining the density distributions. The dynamics are well-described in the mean-field language of coupled Gross-Pitaevskii equations for the order parameters:<sup>22</sup>

$$i\hbar\dot{\Phi}_1 = \left( -\frac{\hbar^2}{2m}\nabla^2 + V_1 + u_1|\Phi_1|^2 + u_{12}|\Phi_2|^2 \right) \Phi_1 \quad (1)$$

and

$$i\hbar\dot{\Phi}_2 = \left( -\frac{\hbar^2}{2m}\nabla^2 + V_2 + V_{\text{hf}} + u_2|\Phi_2|^2 + u_{21}|\Phi_1|^2 \right) \Phi_2 \quad (2)$$

where  $m$  is the mass of the Rb atom,  $V_{\text{hf}}$  is the magnetic field-dependent hyperfine splitting between the two states in the absence of interactions,  $u_i = 4\pi\hbar^2 a_i/m$  and  $u_{ij} = 4\pi\hbar^2 a_{ij}/m$ ,  $|\Phi_i|^2$  is the condensate density, and the intraspecies and interspecies scattering lengths are  $a_i$  and  $a_{ij} = a_{ji}$ . Note that these equations do not allow for species interconversion; they conserve independently the number of atoms in each species.

Considerable theoretical work has already gone into studying ground-state solutions<sup>22</sup> and small-amplitude excitations<sup>23</sup> of the order parameters. We review here the results most relevant to the current experiments. As the total number of atoms trapped increases, the quantum kinetic energy

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term  $\nabla^2\Phi$  becomes less significant, until it is almost negligible for the numbers used in double-condensate experiments. Most theoretical treatments neglect this term, thus making the so-called Thomas-Fermi approximation. In this case, the steady-state, single-component condensate density profile in a parabolic potential will have an inverted paraboloid density profile, with density tapering smoothly from the peak at cloud center to zero at its edge. Due to an accidental degeneracy<sup>4,24</sup> in the Rb-87 scattering system,  $a_1 \approx a_{12} \approx a_2$ . For this special case, one can see by inspection of Eqs. 1 and 2 that the steady-state total density of a two-component BEC will also have an inverted parabola form, even though the relative densities of the components may have intricate structure. It turns out there is a critical value<sup>22</sup> for the interaction term  $a_{12}^c = \sqrt{a_1 a_2}$ . For  $a_{12} > a_{12}^c$ , two components should have little spatial overlap — they should spontaneously separate in the trap, while for  $a_{12} < a_{12}^c$ , there should be a relatively large region of interpenetration. In Rb-87, the scattering lengths are known at the 1% level to be in the proportion  $a_1 : a_{12} : a_2 :: 1.03 : 1 : 0.97$ , with the average of the three being  $55(3) \text{ \AA}$ .<sup>5,24,25</sup>

Since the mutual interaction term is within 1% of its critical value, anything that breaks the symmetry between the two species becomes important. For instance, the fact that  $a_1 > a_2$  means it is energetically favorable for the  $|1\rangle$  atoms to preferentially move towards the lower density region at the periphery of the cloud, forming a spherical shell around  $|2\rangle$ ,<sup>22</sup> but since the difference between  $a_1$  and  $a_2$  is small, even a rather minor vertical offset in the spatial centers of  $V_1$  and  $V_2$  will result in the components' separating up-and-down, rather than radially in-and-out. In steady-state, then, one expects the clouds to be largely, although not completely<sup>26</sup> spatially separated. Further, because the steady-state energetics only modestly favor component separation, one would expect that in the ensuing dynamical behavior, small oscillations about the steady-state configuration<sup>23</sup> will feature a number of “soft” modes, with frequencies low compared to the excitations of the total density.

All of these qualitative theoretical expectations have been borne out in preliminary experiments.<sup>6</sup> Figure 2 shows the density profile<sup>27</sup> of two condensates after they have come to steady-state. In this measurement, the potentials have been offset a distance equal to only 3% of the overall sample size, but the resulting component separation is much larger. Note however that there does remain a region of overlap, which is useful for work described below. The  $|2\rangle$  component is originally created (as described in Section 3.2, below) such that it completely overlaps the  $|1\rangle$  component, and we have observed<sup>6</sup> the time evolution of the component separation as it relaxes to the steady-state, separated condition. During this time, the overall density

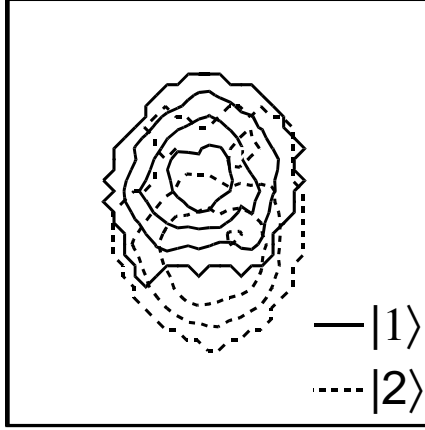


Fig. 2. Contour plot of the expanded<sup>27</sup> density distribution after the two components have undergone separation and their relative motions have damped. The image is 182 microns square. Before expansion the size of the total distribution was 40 by 14 microns. Note that a considerable region of interpenetration remains.

profile is essentially unperturbed. The period of the resulting damped oscillations of the relative positions of the two components is about 30 ms. In comparison, the period of the lowest-order nontrivial excitation of the total density profile is about 25 ms.

If we do not deliberately break symmetry of the confining potentials in the experiment but rather keep them as perfectly overlapped as possible, we see, at early times, the onset of the expected inward-outward component separation. At longer times the inner ball of  $|2\rangle$  atoms does not stay well-centered in the hollow shell of  $|1\rangle$  atoms, but drifts horizontally towards a preferred side. There appears to be an as-yet uncharacterized imperfection in the trap that breaks the radial symmetry of the experimental environment.

The Rubidium-87 mixture is well-suited to a variety of experiments in mixtures of dilute bose gases. There are a host of experimental parameters to explore: the radial:axial aspect ratio of the confining potential (from 3:1 to greater than 1:30, depending on the form of magnetic trap used); the relative number in the two components; and the temperature. The critical temperature of a particular component depends on the number of atoms in that state. Thus, one can arrange for one component to remain in the condensed state while the other one converts from the normal to the condensed state or *vice-versa*. All of these changes will have profound effects on the steady-state configuration and the spectrum of collective excitations. For example, imagine an experiment to study the effects of finite temperature

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on the excitations.<sup>28</sup> At finite temperature, there will be in effect four different co-trapped fluids: normal and condensate fractions of both components. The normal fraction of a given component will find the condensate fraction of its own component twice as repulsive as it finds the condensate fraction of a dissimilar component (due to exchange terms.) Thus if one chooses a potential in which condensate  $|1\rangle$  and condensate  $|2\rangle$  tend to spatially separate, one can have a situation in which the normal fraction of component  $|1\rangle$  tends to preferentially collect at the spatial location of condensate  $|2\rangle$ , and *vice-versa*. What will the collective excitation spectrum look like? If we were to describe these excitations with the word “sound,” what ordinal number should modify it?

## 3. PHASE COHERENCE

In the absence of any applied coupling the 6.8 GHz energy separation between the two states is enormous, but the presence of a oscillating magnetic field tuned close to the hyperfine splitting changes the situation dramatically. An atom can readily be transferred from one state to the other — the difference in energy is absorbed from (or stimulated into) the coherent field of 6.8 GHz photons.<sup>16</sup> That the population transfer resulting from the application of a coupling field is sensitive to the relative phase of the two states is well-known in microwave spectroscopy<sup>29</sup> and NMR and is easy to understand in the case of a single, two-level atom.

### 3.1. Single-Atom Case

The Schrödinger equation<sup>30</sup> for the internal state of a two-level atom (or of any two-level quantum system) in the presence of a coupling drive at frequency  $\omega_{\text{rf}}$  is

$$i\dot{A}_1 = \omega_1 A_1 + \frac{\Omega}{2} e^{i\omega_{\text{rf}} t} A_2 \quad (3)$$

$$i\dot{A}_2 = \omega_2 A_2 + \frac{\Omega^*}{2} e^{-i\omega_{\text{rf}} t} A_1 \quad (4)$$

where  $A_i$  is the probability amplitudes to be in state  $|i\rangle$ ,  $\hbar\omega_i$  is the internal energy of state  $|i\rangle$ , and  $\Omega$  is the amplitude of the coupling. We assume the detuning  $\delta \equiv \omega_{\text{rf}} - \omega_2 + \omega_1$  is small, such that  $|\delta| \ll \omega_{\text{rf}}$ , and concentrate on the case that the coupling drive is off ( $\Omega = 0$ ) most of the time and on only for brief pulses of duration  $\tau$ , such that  $1/\tau \gg |\delta|$ . When we discuss “phase”

in this context, we are talking about the real quantities  $\alpha_1$ ,  $\alpha_2$ , or  $\alpha_{\text{rf}}$  defined in the relations  $A_1 = |A_1|e^{i\alpha_1}$ ,  $A_2 = |A_2|e^{i\alpha_2}$ , and  $\Omega e^{-i\omega_{\text{rf}}t} = |\Omega|e^{i\alpha_{\text{rf}}}$ .

There is one particularly useful special-case solution of Eqs. 3 and 4. A pulse with duration and amplitude such that  $|\Omega|\tau = \frac{\pi}{2}$  is known as a “ $\frac{\pi}{2}$ -pulse.” If the atom is initially in state  $|1\rangle$  with unit probability (*i.e.*,  $|A_1| = 1$  and  $|A_2| = 0$ ), then after the application of the pulse the atom will have equal probability of being in either state (*i.e.*,  $|A_1| = 1/\sqrt{2}$  and  $|A_2| = 1/\sqrt{2}$ ). The  $\frac{\pi}{2}$ -pulse thus creates a 50-50 coherent superposition of the two states, also “writing” a particular initial relative phase. After the pulse is applied, the relative phase evolves at the rate  $\omega_2 - \omega_1$ . The accumulated phase can be measured by a second  $\frac{\pi}{2}$ -pulse.<sup>29</sup> When a  $\frac{\pi}{2}$ -pulse is applied to a two-level system in 50-50 coherent superposition, the population transferred by the pulse from state  $|1\rangle$  to state  $|2\rangle$  is proportional to  $\cos(\alpha_1 - \alpha_2 - \alpha_{\text{rf}})$ , where the phases  $\alpha$  refer to the total phase accumulated during the dwell time  $T$  between the two pulses. One measures the transition probability from state 1 to state 2 by repeating the experiment for different  $T$ ; the result is “Ramsey fringes,” a cosine at the detuning frequency  $\delta$ . This technique is known as the method of separated oscillatory fields<sup>29</sup> or “twin-pulse” spectroscopy.

### 3.2. Application to Order Parameters

Now it becomes clear how we modify Eqs. 1 and 2 to include a coupling field applied to a bose condensate:

$$i\hbar\dot{\Phi}_1 = \left( -\frac{\hbar^2}{2m}\nabla^2 + V_1 + u_1|\Phi_1|^2 + u_{12}|\Phi_2|^2 \right) \Phi_1 + \frac{\hbar\Omega(t)}{2}e^{i\omega_{\text{rf}}t}\Phi_2 \quad (5)$$

and

$$i\hbar\dot{\Phi}_2 = \left( -\frac{\hbar^2}{2m}\nabla^2 + V_2 + V_{\text{hf}} + u_2|\Phi_2|^2 + u_{21}|\Phi_1|^2 \right) \Phi_2 + \frac{\hbar\Omega(t)}{2}e^{-i\omega_{\text{rf}}t}\Phi_1 \quad (6)$$

The probability amplitudes  $A_i$  of Eqs. 3 and 4 have now become field amplitudes  $\Phi_i$ , and the physics has become correspondingly richer. One can still perform experiments, however, that explore simple limits.

In one such experiment, we begin with atoms in the pure  $|1\rangle$ , relaxed to a steady-state, near-pure condensate with spatial distribution described by  $\Phi_0$ . Next we apply a  $\frac{\pi}{2}$ -pulse, transferring half the population to state  $|2\rangle$ . Immediately after the  $\frac{\pi}{2}$ -pulse, the two-state system can be described by  $\Phi_1 = \Phi_0 e^{i\mu_1 t}/\sqrt{2}$ , and  $\Phi_2 = \Phi_0 e^{i\mu_2 t}/\sqrt{2}$ , where  $\mu_1$  and  $\mu_2$ , which differ by about the hyperfine frequency, are the chemical potentials of the two



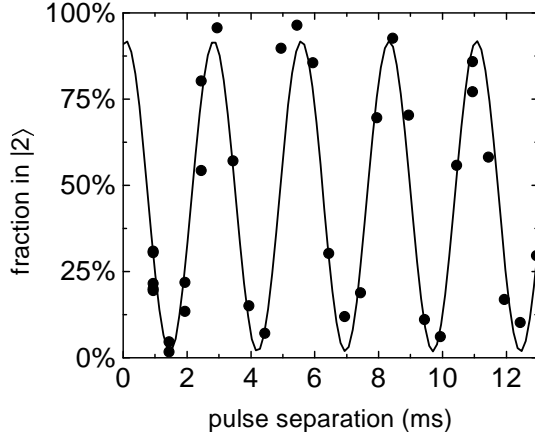


Fig. 3. Population transfer from  $|1\rangle$  to  $|2\rangle$  resulting from twin  $\frac{\pi}{2}$ -pulse coupling pulses, as a function of delay between the two pulses. The first pulse prepares the condensate in an equal superposition of the two states, the second pulse induces further population transfer sensitive to the relative phase that has evolved during the pulse separation time. The coupling drive is detuned from the energy difference between the two condensates by about 360 Hz.

components. We know however that this perfectly overlapping state is not the steady-state of the mixed BEC system; this is in fact the point of departure for the relaxation-to-equilibrium experiments described in Section 2.1. Within a matter of a few tens of milliseconds the components will separate, but on shorter time scales, the condensates don't have time to realize that they are a pair of mutually repulsive quantum fluids. One could equally well describe the system at short times as a single condensate containing about a million atoms, each in an internal, coherent superposition governed by Eqs. 3 and 4. Figure 3 shows the result of applying a second  $\frac{\pi}{2}$ -pulse immediately, and then measuring the final number of atoms in the  $|2\rangle$  state. The high-contrast Ramsey fringes that result are much as one would expect from a similar experiment performed on, for instance, an atomic beam.<sup>29</sup> This “conventional” response of condensates at short times to coupling drives has also been seen in condensate Rabi oscillations.<sup>5,31</sup>

#### 4. COHERENT FLUIDS

If the system is allowed to evolve longer after the first  $\frac{\pi}{2}$ -pulse, so that the components begin to separate, it is no longer very useful to describe the system as a single condensate of atoms in a superposition state. One should

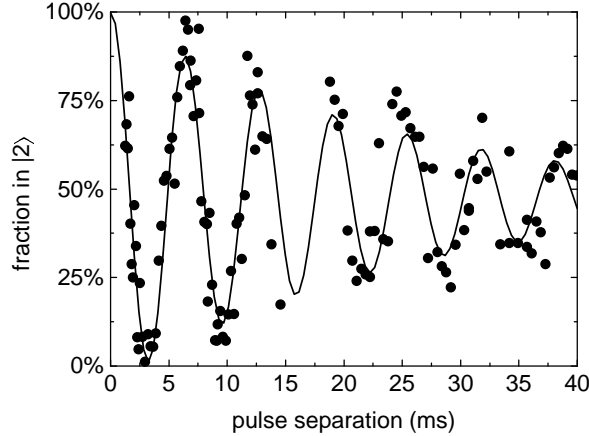


Fig. 4. Population transfer from  $|1\rangle$  to  $|2\rangle$  resulting from twin  $\frac{\pi}{2}$ -pulse coupling pulses, as a function of delay between the two pulses. Results are similar to those shown in Fig. 3, except that over the longer duration the components physically separate — the loss of spatial overlap reduces the contrast ratio of the Ramsey fringes.

think of them instead as distinct fluids evolving their separate ways, albeit (as we shall see) with a well-defined relative phase. The effects of component separation show up clearly in twin-pulse spectroscopy: Figure 4 shows some condensate Ramsey fringes collected over longer periods. The loss of contrast in the fringes is not due to inhomogeneous broadening of the atomic transition, at least not in the conventional sense. Rather, it arises from the dwindling spatial overlap between the two components, which in turn arises from the mutual repulsion of the components, as we saw in Section 2. During component separation the components are of course moving with respect to each other, and therefore there is a gradient in the relative phase across the sample; it is no longer so simple to define a single quantity as the phase difference between the two clouds.

At still longer inter-pulse times, the results of the twin-pulse experiments become once again explainable in terms of a single relative phase. After the component-separation process has gone to completion, the relative motion of the condensates damps, and presumably the gradients of the relative phase vanish as well. As shown in Fig. 2 above, the post-separation clouds preserve a reasonable amount of spatial overlap. As shown schematically in Fig. 5, the application of the second  $\frac{\pi}{2}$ -pulse at this time acts as sort of recombiner on a separated-arm atom-interferometer.<sup>32</sup> Depending on the relative phase between the two components, we can see (Fig. 6) destructive or constructive interference in the region where the components

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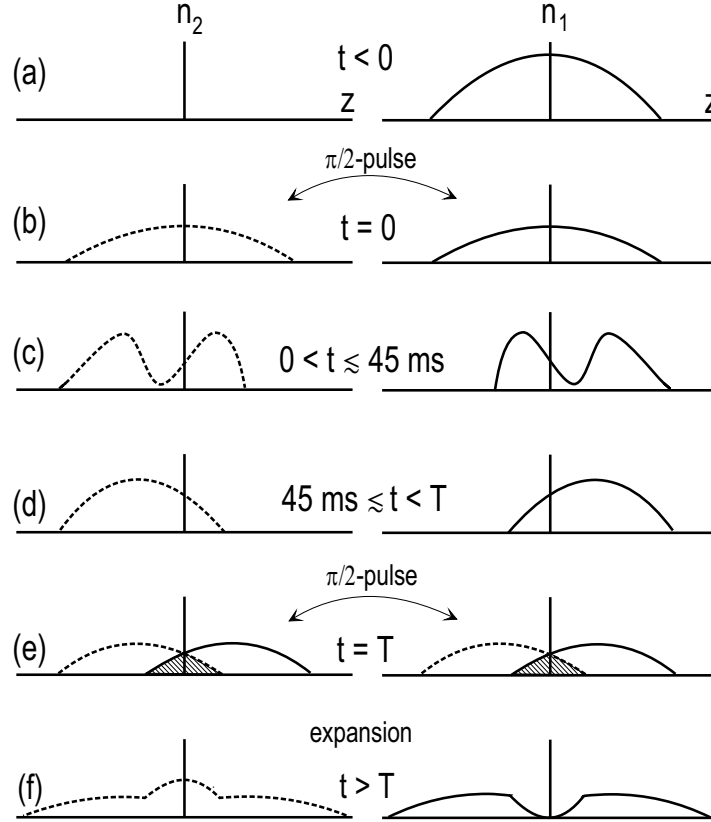


Fig. 5. A schematic<sup>7</sup> of the condensate interferometer. (a) The experiment begins with all of the atoms in condensate  $|1\rangle$  at steady-state. (b) After the first  $\frac{\pi}{2}$ -pulse, the condensate has been split into two components with a well-defined initial relative phase. (c) The components begin to separate in a complicated fashion due to mutual repulsion as well as a  $0.4 \mu\text{m}$  vertical offset in the confining potentials (see also Fig. 3 of Ref. 6). (d) The relative motion between the components eventually damps with the clouds mutually offset but with some residual overlap. Relative phase continues to accumulate between the condensates until (e) at time  $T$  a second  $\frac{\pi}{2}$ -pulse remixes the components; the two possible paths by which the condensate can arrive in one of the two states in the hatched regions interfere. (f) The cloud is released immediately after the second pulse and allowed to expand for imaging. In the case shown, the relative phase between the two states at the time of the second pulse was such as to lead to destructive interference in the  $|1\rangle$  state and a corresponding constructive interference in the  $|2\rangle$  state.

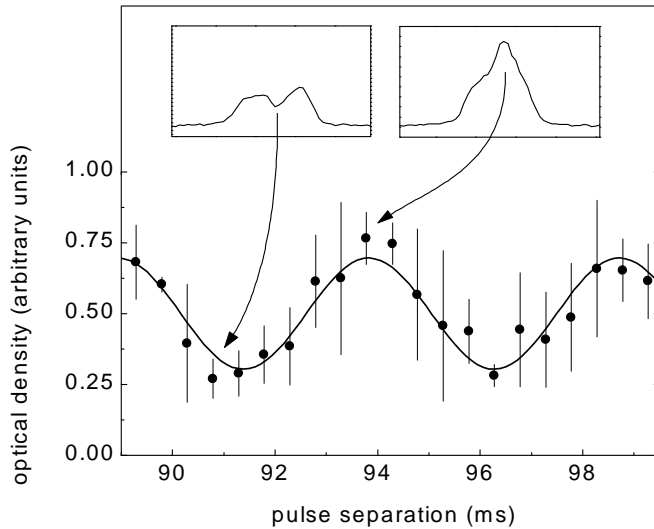


Fig. 6. The value of the condensate density in the  $|2\rangle$  state is extracted at the center of the overlap region (inset) and plotted as a function of  $T$ . Each point represents the average of 4 separate realizations and the thin bars denote the rms scatter in the measured interference for an individual realization. The thick lines are sinusoidal fits to the data, from which we extract the angular frequency  $\mu_2 - \mu_1 - \omega_{\text{rf}}$ .

overlap. Thus, the double-pulse experiment with a long-time between the pulses acts as a condensate interferometer that allows us to determine the relative phase between two largely separated condensate components.

We find the contrast ratio manifest in Fig. 6 quite remarkable. It is evidence that the two components have a well-preserved memory of their relative phase *even after* there has been damping of the external degrees of freedom, with which the internal states are entangled. The phase of the condensate evidently has a robustness one would not expect in a single-particle experiment. This is clearly an area worthy of extensive study – how robust is the relative phase, as a function of temperature, of relative equilibrium separation, of time between pulses? How well can the relative phase survive a “nondestructive” observation,<sup>20</sup> which can take many sequential images of the same condensate? In single-atom interferometry, the connection between the availability of “which-path” information and the loss of coherence has been thoroughly examined.<sup>33</sup> How do these results generalize to condensates?

## 5. ADDITIONAL DIRECTIONS

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### 5.1. Josephson Effects

In the discussion so far the coupling drive was operated in the very short-pulsed mode. If instead it is applied as a weak, continuous drive, it has correspondingly smaller bandwidth, such that it is resonant in certain regions of the condensate, and off-resonant elsewhere. Population transfer can be constrained to occur only within a particular spatial region, for instance right at the overlap region, and can show oscillatory but nonlinear behavior. We are performing experiments<sup>34</sup> in this regime, and are exploring the close analogy<sup>35,36</sup> between the resulting population transfer and a nonlinear Josephson junction.

### 5.2. Number States

In experiments to date, the relative phase appears random if measured more than 150 ms after the condensates are split. This could be due to intrinsic decohering mechanisms, or it could be due to technical instabilities. It is diverting to contemplate, however, creating a condensate pair for which from the very beginning the relative phase is *intrinsically* unpredictable. The conjugate variable to the relative phase is the number difference  $\Delta N = N_1 - N_2$ . For experiments with relatively small total  $N$ , say less than 2000, it may be possible to create states for which  $N_1 \approx N_2 \approx N/2$ , with the uncertainty in  $\Delta N$  less than plus or minus 1 atom. Such a state, analogous to a highly squeezed state of light, will have a completely undefined relative phase and could be useful for performing sub-shotnoise spectroscopy,<sup>37</sup> and illustrates some instructive paradoxes of quantum measurement theory.<sup>38</sup>

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16. A transition from the  $|1\rangle$  state ( $F = 1, m = -1$ ) to the  $|2\rangle$  state ( $F = 2, m = 1$ ) requires two units of angular momentum. The microwave coupling drive must therefore be composed of two photons. As described in Ref. 5, we use a two-photon drive well-detuned from the intermediate  $F = 1, m = 0$  state. For all practical purposes, one may model the coupling drive as a single photon transition with a particular detuning and effective strength.
17. The exception is Ref. 4, for which states  $|1\rangle$  and  $|3\rangle$  were used. In Ref. 4 it was discovered that, due to an accidental degeneracy in scattering lengths, spin-exchange processes are suppressed in Rb-87 (see Ref. 24). In the absence of this degeneracy, one would expect condensate mixtures of different hyperfine levels to have a very short lifetime.
18. The MIT BEC group has confined condensates in optical potentials (Ref. 8) and shown that experiments can be performed on mixtures on the  $m = 1, m = 0$  and  $m = -1$  sublevels of the  $F = 1$  hyperfine state of sodium.
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## Distinguishable Yet Phase-Coherent Condensates

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